

Mathematics philosophy in the Woodbridge School District builds on the belief that children begin school with mathematical intuition and with the ability to think mathematically. School provides a mathematical environment into which come ideas from the broader world and from children's own experiences. Children's relationships with mathematics outside of school are shaped by their experiences with mathematics in school. Through their experiences at Beecher Road School, children will develop an understanding of the world as a mathematical environment.

- Students' experiences in school will promote a positive disposition toward mathematics.
- Students' math programs will recognize and build upon the intuition and awareness with which they begin school.
- Students will have opportunities and will be encouraged to construct their own mathematical understandings.
- Students will use appropriate math tools strategically.
- Students will make use of problems and persevere in solving them.
- Students will reason abstractly and quantitatively.
- Students will construct viable arguments and critique the reasoning of others.
- Students will apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Students will attend to detail and evaluate the reasonableness of their results.

Supporting Your Child's Math Education at Home

- Encourage your children to think of themselves as mathematicians. Acknowledge your own enthusiasm for mathematics.
- Try not to allow your children to convince themselves that they are unable to do math. Their confidence is critical to their success in mathematics.
- If your children don't know their math facts, work with them regularly at home, in the car, or at other appropriate times to memorize these facts. They provide the foundation for work in mathematics.
- Remember that mathematics instruction is different now than it was when you were in school. At first, the language and approach may be uncomfortable for you. Try not to struggle to assist when you don't understand what is being asked. Instead, encourage your children to formulate questions that they can bring to the teacher.
- There are many ways to incorporate math into daily life. While shopping you may ask questions like, "If one box of cereal costs \$3.00, how much money would two boxes cost?" While driving, you could say, "We're at exit 24 and we need to go to exit 29. How many exits to go?" At home, you can discuss fractions when you are cooking by looking at the marks on a measuring cup. You can reinforce time concepts by asking your child to tell you when the clock says a certain time. As you become more comfortable using math language and engaging your children in mathematical discussions you will find these kinds of conversations are easy to do and can be fun as well!

Common Core State Standards for Mathematics¹

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." The Common Core Standards are a substantial answer to that challenge.

Fewer standards are no substitute for *focused standards*. Merely achieving fewer standards would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

The Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, the origin of a particular rule or why a specific mathematical statement is true. The Common Core State Standards for Mathematics (CCSSM) include two types of standards, one for *mathematical content* (what students know about math) and one for *mathematical practice* (how students are able to apply and extend math principles). The two are linked together while students are learning.

The *Standards for Mathematical Practice* describe ways in which developing student practitioners of the discipline of mathematics ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the

¹ http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

elementary, middle and high school years. There are eight Standards for Mathematical Practice:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

The *Standards for Mathematical Content* are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who have a flexible base from which to work are able to:

- consider analogous problems
- represent problems coherently
- justify conclusions
- apply the mathematics to practical situations
- use technology mindfully to work with the mathematics
- explain the mathematics accurately to other students
- step back for an overview
- deviate from a known procedure to find a shortcut.

At Beecher Road School, teachers and math coordinators work collaboratively to design curriculum and assessments and to analyze student data to inform instruction. The CCSSM have led to increased rigor in the mathematics curriculum, raising the bar for all students.

Assessments

Another result of the new math standards is new common assessments. Connecticut is a member of the SMARTER Balanced Assessment Consortium.

SMARTER Balanced is a state-led consortium working to develop next-generation assessments that measure student progress toward college and career readiness and that are aligned to the Common Core State Standards. The work of the Consortium is guided by the belief that a high-quality assessment system can provide resources and tools for teachers and schools to improve instruction and help students succeed.

The SMARTER Balanced assessment system capitalizes on the precision and efficiency of computer adaptive testing (CAT) for both mandatory summative assessments and optional interim assessments.

CAT offers teachers and schools a more accurate way than previous pencil-and-paper assessments to evaluate student achievement and readiness for college and careers, and to measure growth over time. Based on student responses, the computer program adjusts the difficulty of questions throughout the assessment. For example, a student who answers a question correctly will receive a more challenging item, while an incorrect answer generates an easier question. By adapting to each student’s abilities, these assessments quickly identify which skills students have mastered.

An Overview of Mathematics in Kindergarten

In kindergarten, students focus on representing and comparing whole numbers, addition and subtraction, and identifying and describing shapes. Activities in these areas will include:

- Counting to 100 by ones and by tens
- Counting forward beginning from a given number
- Writing numbers from 0 - 20
- Counting to tell the number of objects and comparing the quantities of two groups of objects
- Comparing two numbers between 1 and 10 to identify which is greater or less than the other
- Understanding addition as putting together and adding to, and understanding subtraction as taking apart and taking from
- Fluently adding and subtracting within 5
- For any number from 1 to 9, finding the number that makes 10 when added to a given number
- Composing and decomposing numbers from 11 to 19 into ten ones and some further ones
- Solving addition and subtraction word problems
- Describing measurable attributes of objects, such as length or weight
- Classifying objects into given categories
- Identifying and describing shapes

An Overview of Mathematics in Grade 1

In Grade 1 students focus on developing an understanding of addition and subtraction, whole number relationships and place value, and linear measurement. Students also reason about attributes of shapes. Activities in these areas will include:

- Using addition and subtraction within 20 to solve word problems
- Adding and subtracting within 20, demonstrating fluency for addition and subtraction within 10
- Understanding the meaning of the equal sign, and determining if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.
- Understanding that the two digits of a two-digit number represent amounts of tens and ones
- Adding a two-digit number and a one-digit number, adding a two-digit number and a multiple of 10, and subtracting multiples of ten from multiples of ten
- Ordering objects by length
- Measuring the lengths of objects using a shorter object as a unit of measure
- Telling and writing time in hours and half-hours using analog and digital clocks.
- Organizing, representing, and interpreting data
- Reasoning with shapes and their attributes
- Dividing circles and rectangles in two and four equal shares

An Overview of Mathematics in Grade 2

In Grade 2, students extend their understanding of the base-ten system and begin to develop fluency with addition and subtraction within 100. Second graders recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools. In addition, students describe and analyze shapes by examining their sides and angles. Students also investigate, describe, and reason about shapes. Activities in these areas will include:

- Using addition and subtraction within 100 to solve one- and two-step word problems
- Fluently adding and subtracting within 20
- Understanding that the three digits of a three-digit number represent amounts of hundreds, tens, and ones
- Reading and writing numbers to 1000 and skip-counting by fives, tens, and hundreds
- Comparing two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons
- Fluently adding and subtracting within 100 using various strategies
- Measuring the length of an object by selecting and using various tools
- Estimating lengths using units of inches, feet, centimeters, and meters
- Telling and writing time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
- Solving word problems involving money
- Representing and interpreting data using picture and bar graphs
- Recognizing and drawing shapes having specified attributes
- Partitioning circles and rectangles into two, three, or four equal shares and describing the shares using the words halves, thirds, and fourths
- Partitioning a rectangle into rows and columns of same-size squares and counting to find the total number of them

An Overview of Mathematics in Grade 3

In grade 3, students will continue to build their concept of numbers, developing an understanding of fractions as numbers. They will learn the concepts behind multiplication and division and apply problem-solving skills and strategies for multiplying and dividing numbers up through 100 to solve word problems. Students will also make connections between the concept of the area of a rectangle and multiplication and addition of whole numbers. Activities in these areas include:

- Fluently adding and subtracting within 1000 using strategies and algorithms based on place value
- Understanding and explaining what it means to multiply or divide numbers
- Fluently multiplying and dividing all one-digit numbers
- Multiplying one-digit numbers by multiples of 10 (such as 20, 30, 40)
- Solving two-step word problems using addition, subtraction, multiplication, and division
- Understanding the concept of area
- Relating the measurement of area to multiplication and division
- Understanding fractions as numbers
- Understanding and identifying a fraction as a number on a number line
- Comparing the size of two fractions
- Expressing whole numbers as fractions and identifying fractions that are equal to whole numbers (for example, recognizing that $\frac{3}{1}$ and 3 are the same number)

- Measuring weights and volumes and solving word problems involving these measurements
- Representing and interpreting data

An Overview of Mathematics in Grade 4

In grade four, your child will use addition, subtraction, multiplication, and division to solve word problems, including problems involving measurement of volume, mass, and time. Students will continue to build their understanding of fractions- creating equal fractions, comparing the size of fractions, adding and subtracting fractions, and multiplying fractions by whole numbers. They will also start to understand the relationship between fractions and decimals. Activities in these areas will include:

- Fluently adding and subtracting multi-digit whole numbers using the standard algorithm
- Solving multi-step word problems, including problems involving measurement and converting measurements from larger to smaller units.
- Multiplying and dividing multi-digit numbers
- Extending understanding of fractions by comparing the size of two fractions with different numerators and different denominators
- Creating equal fractions
- Adding and subtracting fractions with the same denominator
- Building fractions from smaller fractions ($\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$)
- Connecting addition and subtraction of whole numbers to multiplying fractions by whole numbers
- Representing and interpreting data
- Converting fractions with denominators of 10 or 100 into decimals
- Locating decimals on a number line
- Comparing decimals and fractions using the symbols $<$ (less than), $>$ (greater than) and $=$ (equal to)

An Overview of Mathematics in Grade 5

In grade five, students will build their understanding of the place value system by working with decimals up to the hundredths place. Students will also add, subtract, and multiply fractions, including fractions with unlike denominators. They will continue to expand their geometry and measurement skills, learning the concept of volume and measuring the volume of a solid. Activities in these areas will include:

- Fluently multiplying multi-digit whole numbers using the standard algorithm
- Dividing numbers with up to four digits by two digit numbers using strategies based on place value
- Using exponents to express powers of 10
- Reading, writing, and comparing decimals to the thousandths place
- Adding, subtracting, multiplying, and dividing decimals to the hundredths place
- Writing and interpreting mathematical expressions using symbols such as parentheses. For example, “*add 8 and 7, then multiply by 2*” can be written as $2 \times (8+7)$.
- Adding and subtracting fractions with unlike denominators by converting them to fractions with matching denominators

- Multiplying fractions by whole numbers and other fractions
- Dividing fractions by whole numbers and whole numbers by fractions
- Solving real world problems involving multiplication and division of fractions
- Analyzing and determining relationships between numerical patterns
- Measuring volume using multiplication and addition

An Overview of Mathematics in Grade 6

In grade six, students will learn the concept of rates and ratios and use these tools to solve word problems. Students will work on quickly and accurately dividing multi-digit whole numbers and adding, subtracting, multiplying, and dividing multi-digit decimals. Students will extend their previous work with fractions and decimals to understand the concept of rational numbers—any number that can be made by dividing one integer by another, such as $\frac{1}{2}$, 0.75, or $\frac{1}{2}$. Students will also learn how to write and solve equations—mathematical statements using symbols, such as $20+x = 35$ —and apply these skills in solving multi-step word problems. Activities in these areas will include:

- Fluently dividing multi-digit whole numbers using the standard algorithm
- Fluently adding, subtracting, multiplying and dividing multi-digit decimals using the standard algorithm
- Understanding and applying the concepts of ratios and unit rates, and using the correct language to describe them (for example, the ratio of wings to beaks in a flock of birds is 2 to 1, because for every 2 wings there is 1 beak)
- Building on knowledge of multiplication and division to divide fractions by fractions
- Understanding that positive and negative numbers are located on opposite sides of 0 on a number line
- Using pairs of numbers, including negative numbers, as coordinates for locating or placing a point on a graph
- Writing and determining the value of expressions with whole-number exponents (such as $15 + 3^2$)
- Identifying and writing equivalent mathematical expressions by applying the properties of operations. For example, recognizing that $2(3+x)$ is the same as $6+2x$
- Understanding that solving an equation such as $2+x = 12$ means answering the question, “*What number does x have to be to make this statement true?*”
- Representing and analyzing the relationships between independent and dependent variables
- Solving problems involving area and volume

Problem Solving Trajectories

	Result Unknown	Change Unknown	Start Unknown
Add To	<p><i>A</i> bunnies sat on the grass. <i>B</i> more bunnies hopped there. How many bunnies are on the grass now?</p> $A + B - \square$	<p><i>A</i> bunnies were sitting on the grass. Some more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies hopped over to the first <i>A</i> bunnies?</p> $A + \square - C$	<p>Some bunnies were sitting on the grass. <i>B</i> more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies were on the grass before?</p> $\square + B - C$
Take From	<p><i>C</i> apples were on the table. I ate <i>B</i> apples. How many apples are on the table now?</p> $C - B - \square$	<p><i>C</i> apples were on the table. I ate some apples. Then there were <i>A</i> apples. How many apples did I eat?</p> $C - \square - A$	<p>Some apples were on the table. I ate <i>B</i> apples. Then there were <i>A</i> apples. How many apples were on the table before?</p> $\square - B - A$
	Total Unknown	Both Addends Unknown ¹	Addend Unknown ²
Put Together /Take Apart	<p><i>A</i> red apples and <i>B</i> green apples are on the table. How many apples are on the table?</p> $A + B - \square$	<p>Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase?</p> $C - \square + \square$	<p><i>C</i> apples are on the table. <i>A</i> are red and the rest are green. How many apples are green?</p> $A + \square - C$ $C - A - \square$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	<p><i>"How many more?"</i> version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many more apples does Julie have than Lucy?</p> <p><i>"How many fewer?"</i> version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many fewer apples does Lucy have than Julie?</p> $A + \square - C$ $C - A - \square$	<p><i>"More"</i> version suggests operation. Julie has <i>B</i> more apples than Lucy. Lucy has <i>A</i> apples. How many apples does Julie have?</p> <p><i>"Fewer"</i> version suggests wrong operation. Lucy has <i>B</i> fewer apples than Julie. Lucy has <i>A</i> apples. How many apples does Julie have?</p> $A + B - \square$	<p><i>"Fewer"</i> version suggests operation. Lucy has <i>B</i> fewer apples than Julie. Julie has <i>C</i> apples. How many apples does Lucy have?</p> <p><i>"More"</i> version suggests wrong operation. Julie has <i>B</i> more apples than Lucy. Julie has <i>C</i> apples. How many apples does Lucy have?</p> $C - B - \square$ $\square + B - C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Table 3: Multiplication and division situations

	$A \times B = \square$	$A \times \square = C$ and $C \div A = \square$	$\square \times B = C$ and $C \div B = \square$
Equal Groups of Objects	<p>Unknown Product</p> <p>There are A bags with B plums in each bag. How many plums are there in all?</p>	<p>Group Size Unknown</p> <p>If C plums are shared equally into A bags, then how many plums will be in each bag?</p>	<p>Number of Groups Unknown</p> <p>If C plums are to be packed B to a bag, then how many bags are needed?</p>
Arrays of Objects	<p>Unknown Product</p> <p>There are A rows of apples with B apples in each row. How many apples are there?</p>	<p><i>Equal groups language</i></p> <p>Unknown Factor</p> <p>If C apples are arranged into A equal rows, how many apples will be in each row?</p>	<p>Unknown Factor</p> <p>If C apples are arranged into equal rows of B apples, how many rows will there be?</p>
	<p>Unknown Product</p> <p>The apples in the grocery window are in A rows and B columns. How many apples are there?</p>	<p><i>Row and column language</i></p> <p>Unknown Factor</p> <p>If C apples are arranged into an array with A rows, how many columns of apples are there?</p>	<p>Unknown Factor</p> <p>If C apples are arranged into an array with B columns, how many rows are there?</p>
Compare	<p>Larger Unknown</p> <p>A blue hat costs $\\$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?</p>	<p>$A > 1$</p> <p>Smaller Unknown</p> <p>A red hat costs $\\$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?</p>	<p>Multiplier Unknown</p> <p>A red hat costs $\\$C$ and a blue hat costs $\\$B$. How many times as much does the red hat cost as the blue hat?</p>
	<p>Smaller Unknown</p> <p>A blue hat costs $\\$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?</p>	<p>$A < 1$</p> <p>Larger Unknown</p> <p>A red hat costs $\\$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?</p>	<p>Multiplier Unknown</p> <p>A red hat costs $\\$C$ and a blue hat costs $\\$B$. What fraction of the cost of the blue hat is the cost of the red hat?</p>

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B , so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

Multiplicative Compare problems appear first in Grade 4, with whole-number values for A , B , and C , and with the “times as much” language in the table. In Grade 5, unit fractions language such as “one third as much” may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., “A red hat costs A times as much as the blue hat” results in the same comparison as “A blue hat costs $1/A$ times as much as the red hat,” but has a different subject.